South Otterington Church of England Primary School

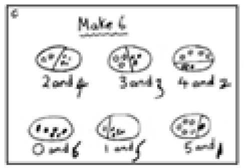
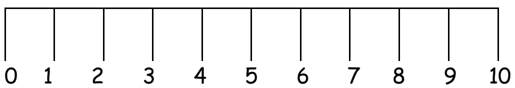
Mathematics Mental and Written Calculation Policy

Mental and written calculation methods should be taught alongside each other throughout the entirety of this progression. When teaching children to calculate emphasis should be placed on choosing and using the method that is most efficient. If a child can complete a calculation mentally or with jottings, they should not be expected to complete a written algorithm. Whilst no longer part of the statutory curriculum, children should also be taught when and how to use a calculator appropriately.

You may find that resources from www.ncetm.org.uk are helpful, particularly the suite of videos which include examples of some of the approaches suggested in this documentations in practice in the classroom. These can be found at https://www.ncetm.org.uk/resources/40529

If you are looking for support in exploring the rationale behind teaching children to use written calculations and the necessary skills, knowledge and understanding that will underpin this work may find the document 'Teaching Written Calculations: Guidance for teachers at Key Stages One and Two' helpful. This and many other documents produced through the National and Primary strategies can be found in the e-library of the STEM centre at http://www.nationalstemcentre.org.uk/elibrary/

ADDITION

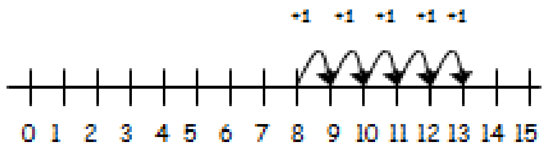
Guidance	**Examples**	
Stage 1: Recording and developing mental pictures • Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They experience practical calculation opportunities using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc. They develop ways of recording calculations using pictures, etc.	**Stage 1**   **One and one, two more**  **makes one, two three."** There are 3 people on the bus. Another person gets on. How many now? 	Initially recording of calculating should be done by adults to model what children have done in pictures, symbols, numbers and words. Over time there should be an expectation that children will also become involved in the recording process. <u>Whilst cameras are an excellent way of keeping a record of what children have done, they are not a substitute for the modelling of different ways of recording calculation procedures.</u>
Stage 2: Progression in the use of a number line • To help children develop a sound understanding of numbers and to be able to use them confidently in calculation, there needs to progression in their use of number tracks and number lines	**Stage 2** Children should experience a range of representations of number lines, such as the progression listed below. Number track | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Number line, all numbers labelled 	**Additional 'number lines' - The bead string and hundred square** •A hundred square is an efficient visual resource to support adding on in ones and tens and is an extension to the number track that children have experienced previously. 8 + 2 = 10

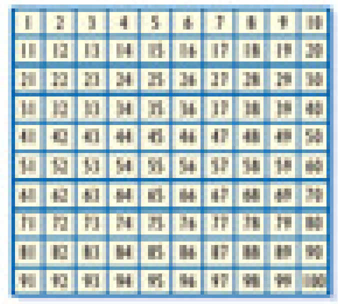
The labelled number line

- Children begin to use numbered lines to support their calculations counting on in ones.
- They select the biggest number first i.e. 8 and count on the smaller number in ones.

- Number line, 5s and 10s labelled
- Number line, 10s labelled
- Number lines, marked but unlabelled

8 + 5 = 13

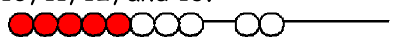




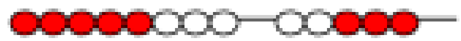
Different orientations of the 100 square help children transfer their skills and understanding between similar representations.

•Along with the number line, bead strings can be used to illustrate addition. Eight beads are counted out, then the two beads. Children count on from eight as they add the two beads e.g. starting at 8 they count 9 then 10 as they move the beads.

•Eight beads are counted out, then the five. Children count on from eight as they add the five e.g. starting at 8 they count 9, 10, 11, 12, and 13.



8 + 5 = 13



Stage 3: The empty number line as a representation of a mental strategy

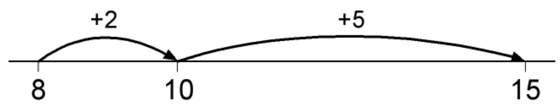
NB It is important to note that the empty number line is intended to be a representation of a mental method, not a written algorithm (method). Therefore the order and size (physical and numerical) of the jumps should be expected to vary from one calculation to the next.

- The mental methods that lead to column addition generally involve partitioning.
- <u>Children need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps.</u>
- The empty number line helps to record the steps on the way to calculating the total.

Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10.

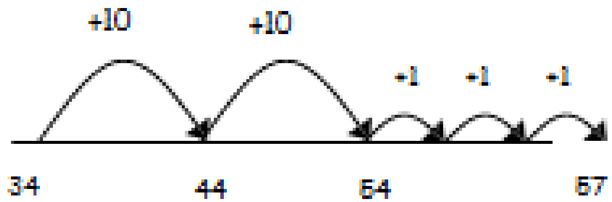
8 + 7 = 15

Seven is partitioned into 2 and 5; 2 creating a number bond to 10 with the 8 and then the 5 is added to the 10.



First counting on in tens and ones.

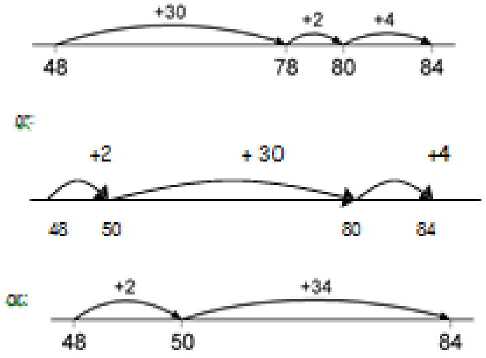
34 + 23 = 57



This develops in efficiency, alongside children's confidence with place value.

Counting on in multiples of 10.

48 + 36 = 84



These examples show how children should be taught to use jumps of different sizes, and completed in an order that is most helpful depending on the numbers they are calculating with. **This reinforces that this is a visual representation of a mental method and not a written alogrithm.**

Stage 4: Partitioning into tens and ones to lead to a formal written method

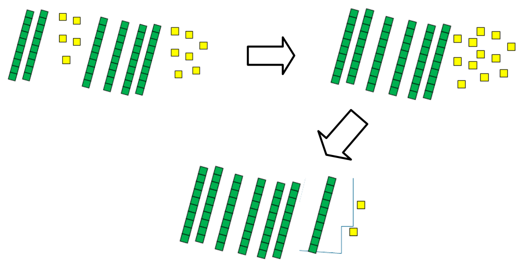
- The next stage is to record mental methods using partitioning into tens and ones separately.

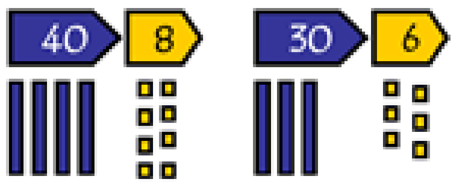
- Add the tens and then the ones to form partial sums and then add these partial sums.

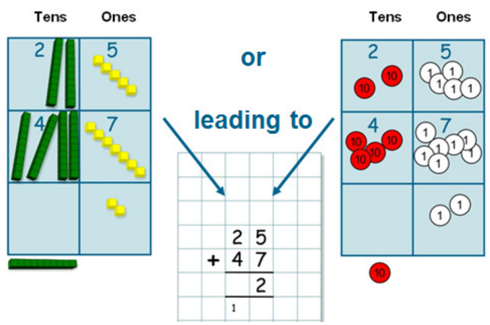
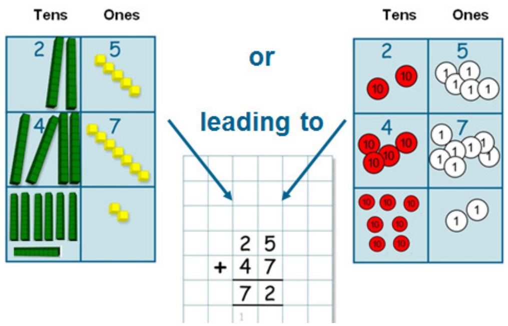
Stage 3

Children should use a range of practical apparatus (place value cards, straws, place value counters) to complete TU + TU. They partition the number into tens and ones before adding the numbers together, finding the total.
There should be progression through this selection of apparatus. Once using abstract representations teachers will start with straws, bundled into 10s and singularly.
Children see 10 straws making one bundle and can be

15 + 47

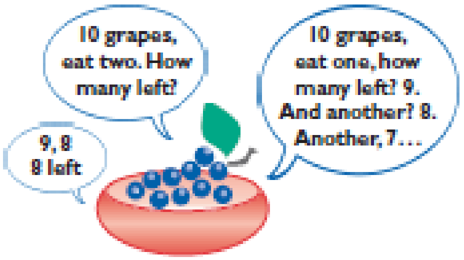


• Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens. This also links to mental methods. • This method can be extended for TU + HTU and HTU + HTU and beyond; as well as cater for the addition of decimal numbers.	involved in bundling and unbundling. This then progresses to the use of Dienes (or similar) where 10s are clearly ten ones but cannot be separated in the same way. Once children are able to use these with understanding, they will progress to the use of place value cards and place value counters which are a further abstraction of the concept of number. Money should also be used (1ps, 10ps and £1) as place value equipment to help children develop their understanding of a range of representations. Progress through these manipulatives should be guided by understanding not age or year group. 48 + 36  40 + 30 = 70 8 + 6 = 14 70 + 14 = 84 Cuisenaire can also be used to support this step, especially when crossing the tens barrier with ones. When this occurs, children should use the term 'exchange' to describe converting ten ones into one ten.	Children may make these jottings to support their calculation. 47 + 76 40 + 70 = 110 **or** 7 + 6 = 13 7 + 6 =13 40 + 70 = 110 110+ 13 = 123 110 + 13 = 123 **or** 47 + 70 = 117 117 + 6 = 123
Stage 5 – Using Dienes/place value counters alongside columnar written method • To ensure the statutory final written method is grounded in understanding, this stage connects the practical equipment to the formal written method using a similar and transferrable layout. • Children first experience the practical version of column addition and when confident in	**It may be appropriate to teach children the process with numbers that they would be expected to calculate mentally or with jottings. This is to aid with the practicalities of the use of such equipment. However this should be the exception rather than the rule so children see a clear purpose for learning a new method for calculating.** In this example	

explaining this, including exchanging when crossing the tens barrier with units, they record the written method alongside. • Ideally children will experience this stage with a variety of practical equipment to make sure their understanding is embedded and transferrable between representations. • Children may learn more from experiencing the inefficiency of not starting with column with least significant value rather than being 'told' where to start.	25 + 47 =   Whilst these images show the total existing alongside the two numbers being added, it may be more representative to 'drag' the manipulatives down to the totals box, leaving the written numbers as a reminder of what was originally there. Another way of representing this is like this	Represented in place value columns and rows. Starting adding with the 'least significant digit' When the tens barriers is crossed in the 'units' exchange then takes place. 5 + 7 = 12 (1 ten + 2 units) Because of the exchange we can know see that this ten belongs in the tens column and is carried there to be included in the total of that column. The tens are then added together 20 + 40 + 10 = 70, recorded as 7 in the tens column This method can be used within an intervention program and involves less movement of equipment
Stage 5: Compact column method • In this method, recording is reduced further. Carried digits are recorded, using the words 'carry ten' or 'carry one hundred' etc., according to the value of the digit. • The carried unit MUST be positioned above the top line of the answer area, this allows	**Stage 5** H T U 2 5 8 $+ \; {}_1 8 {}_1 4$ 3 4 2	

the children to • Later the method is extended when adding more complex combinations such as three two-digit numbers, two three-digit numbers, and problems involving several numbers of different sizes.	**Column addition remains efficient when used with larger whole numbers and once learned, is quick and reliable.** Th H T U 4 5 8 3 + $_1$4$_1$4 5 5 0 2 8	

SUBTRACTION

Guidance	Examples	
Stage 1: Recording and developing mental pictures •Children are encouraged to develop a mental picture of the calculation in their heads. They experience practical activities using a variety of equipment and develop ways to record their findings including models and pictures. • The 'difference between' is introduced through practical situations and images.	**Stage 1**  There are four children in the home corner. One leaves. How many are left?	Initially recording of calculating should be done by adults to model what children have done in pictures, symbols, numbers and words. Over time there should be an expectation that children will also become involved in the recording process. <u>Whilst cameras are an excellent way of keeping a record of what children have done, they are not a substitute for the modelling of different ways of recording calculation procedures.</u>

Stage 2: Progression in the use of a number line

- Finding out how many items are left after some have been 'taken away' is initially supported with a number track followed by labelled, unlabelled and finally empty number lines, as with addition.

The labelled number line

- The labelled number line, linked with previous learning experiences, is used to support calculations where the result is less objects (i.e. taking away) by counting back.

Difference between

- The number line should also be used to make comparisons between numbers, to show that 6 – 3 means the '**difference in value** between 6 and 3' or the 'difference between3 and 6' and how many jumps they are apart.

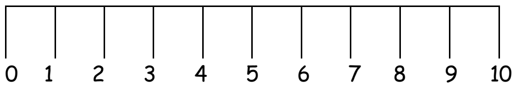
Stage 2

Children should experience a range of representations of number lines, such as the progression listed below.

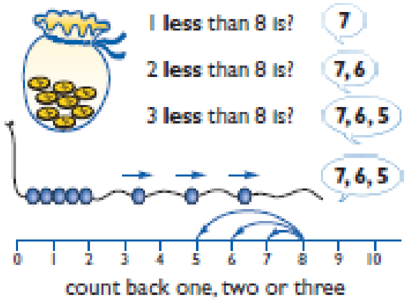
Number track

1	2	3	4	5	6	7	8	9	10

Number line, all numbers labelled

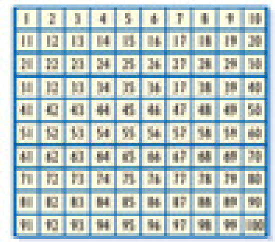


- Number line, 5s and 10s labelled
- Number line, 10s labelled
- Number lines, marked but unlabelled



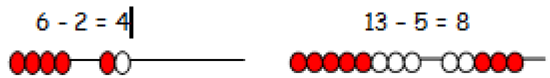
Additional 'number lines' - The bead string and hundred square

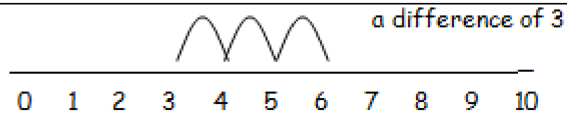
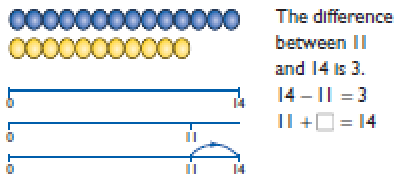
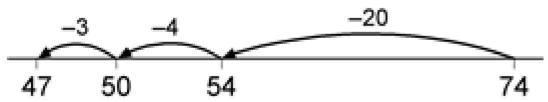
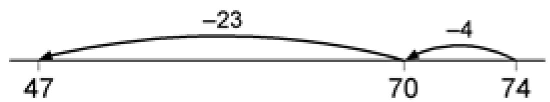
A hundred square is an efficient visual resource to support counting on and back in ones and tens and is an extension of the number track which children have experienced previously

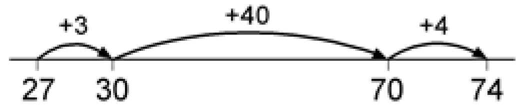
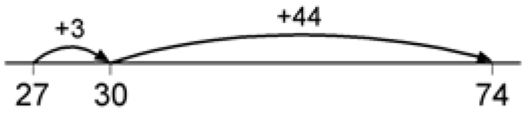
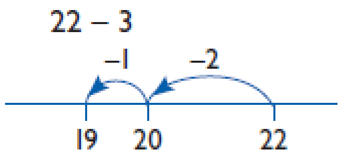
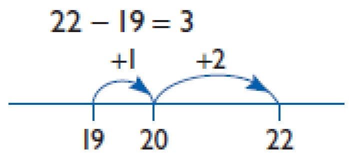


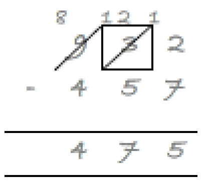
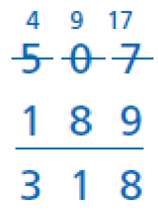
Different orientations of the 100 square help children transfer their skills and understanding between similar representations.

- Bead strings can be used to illustrate subtraction. 6 beads are counted and then the 2 beads taken away to leave 4.

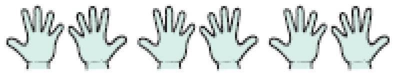


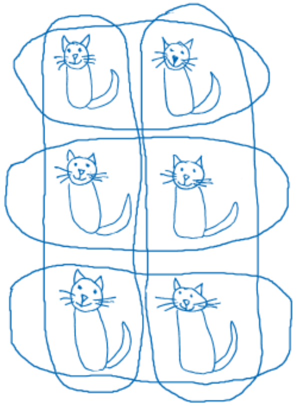
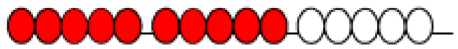
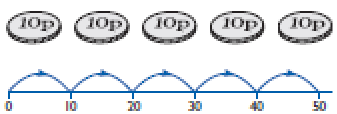
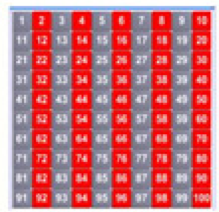
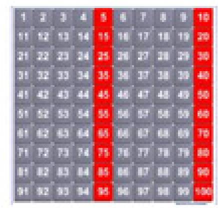
	  	
Stage 3: The empty number line as a representation of a mental strategy **NB It is important to note that the empty number line is intended to be a representation of a mental method, not a written algorithm (method). Therefore the order and size (physical and numerical) of the jumps should be expected to vary from one calculation to the next.** **Finding an answer by COUNTING BACK** • **Counting back** is a useful strategy when the context of the problem results in there being less e.g. Bill has 15 sweets and gives 7 to his friend Jack, how many does he have left? As in addition, children need to be able to partition numbers e.g. the 7 is partitioned into 5 and 2 to enable counting back to 10. • The empty number line helps to	Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10. 15 - 7 = 8 The seven is partitioned into 5 (to allow count back to 10) and two.  74 - 27 = 47 worked by counting back:  	The steps may be recorded in a different order:  or combined  These examples show how children should be taught to use jumps of different sizes, and completed in an order that is most helpful depending on the numbers they are calculating with. **This reinforces that this is a visual representation of a mental method and not a written alogrithm.**

record or explain the steps in mental subtraction. • A calculation like 74 – 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is a useful way of modelling processes such as bridging through a multiple of ten.		
Using an empty number line **Finding an answer by COUNTING ON** • The steps can also be recorded by **counting on** from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47 (shopkeeper's method). This is a useful method when the context asks for comparisons e.g. how much longer, how much smaller; for example: Jill has knitted 27cm of her scarf, Alex has knitted 74cm. How much longer is Alex's scarf? • After practice of both, examples like this will illustrate how children might choose when it is appropriate to count on or back. This also helps to reinforce addition and subtraction as inverses and the links between known number facts.	74 - 27 =  The 'jumps' should be added, either mentally or with jottings according to confidence, beginning with the largest number e.g. 40 + 4 + 3. or    	

Stage 4: Compact method • Finally children complete the compact columnar subtraction as the most efficient form. • Once children are confident with HTU – HTU, this should be extended to four digit subtract four digit calculations.	563 – 246 = 317 5 1 563 246 317 932 - 457 becomes  Answer: 475	 Children may find it more helpful to present their exchanges like this to keep the numbers clear. Schools should decide on a consistent approach to modelling this format and encourage children to be consistent with their layout to reduce error rates. However if individual children use slightly different locations for their representations of exchange but understand and can explain what they are doing and why, the placement is not important.

MULTIPLICATION

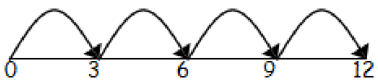
Guidance	**Examples**	
Stage 1: Recording and developing mental images • Children will experience equal groups of objects. They will count in 2s and 10s and begin to count in 5s. • They will experience practical calculation opportunities involving equal sets or groups using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc. • They develop ways of recording calculations using pictures, etc. • They will see everyday versions of arrays, e.g. egg boxes, baking trays, ice cube trays, wrapping paper etc. and use this in their learning answering questions such as; 'How many eggs would we need to fill the egg box? How do you know?' • Children will use repeated addition to carry out multiplication supported by the use of counters/cubes.	**Stage 1**  $2 + 2 + 2 + 2 + 2 = 10$  $5 + 5 + 5 + 5 + 5 + 5 = 30$ $5 \times 6 = 30$  2 groups of 3 are 6 (3 + 3) 3 groups of 2 are 6 (2 + 2 + 2)  4 lots of 3 are 12 3 lots of 4 are 12	Initially recording of calculating should be done by adults to model what children have done in pictures, symbols, numbers and words. Over time there should be an expectation that children will also become involved in the recording process. Whilst cameras are an excellent way of keeping a record of what children have done, they are not a substitute for the modelling of different ways of recording calculation procedures.

	 Children should use pictorial representations and may use rings to show e.g. 3 groups of 2 and 2 groups of 3 introducing the commutative law of multiplication	
Stage 2: The bead string, number line and hundred square • Children continue to use repeated addition to carry out multiplication tasks and represent their counting on a bead string or a number line. • On a bead string, children count out three lots of 5 then count the beads altogether. • On a number line. Children count on in groups of 5. • These models illustrate how multiplication relates to repeated addition.	**Stage 2** 3 lots of 5 5 + 5 + 5 = 15   10p + 10p + 10p + 10p + 10p = 50p 10p × 5 = 50p 5 hops of 10	Children begin pattern work on a 100 square to help them begin to recognise multiples and rules of divisibility.   **Multiples of 2** **Multiples of 5** Children regularly sing songs, chant and play games to reinforce times tables facts and their associated patterns.

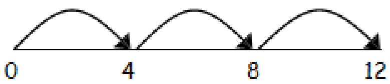
Stage 3: Number line

It is important to be able to visualise multiplication as a rectangular array. This helps children develop their understanding of the commutative law i.e. 3 x 4 = 4 x 3

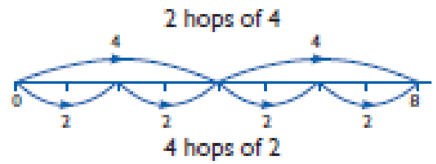
Stage 3



4 lots of 3 = 12



3 lots of 4 =12



For more direct comparison, this could then be demonstrated on a single number line as appropriate.

Stage 4: The Grid Method

- This is the first exposure to the distributive law of multiplication and children should be given plenty of opportunity to explore this
- Children will partition arrays in a variety of helpful ways which are not necessarily the ways in which they will eventually partition them to be in line with formal written methods
- The link between arrays and the grid method should be made clear to children by the use of place value apparatus such as place value counters and Dienes.
- The TU number is partitioned e.g. 13 becomes 10 and 3 and each part of the number is them multiplied by 4.

Stage 4

x	**10**	**3**
4	40	12

40 + 12 = 52

Reinforce the partitioning of the 2-digit numbers into tens and units.

Childern to then move onto TU X TU using the same process but adding an additional row.

Knowing 5 and 2 x tables and being able to add, I can partition this array to use these facts to work.
5 x 5 = 25, 5 x 3 = 15, 5 x 2 = 10
2 x 3 = 6
7 x 8 = 25 + 15 + 10 + 6 = 56

Two-digit by two-digit products using the grid method (TU x TU)

- Having calculated the sections of the grid, children will decide whether to add the rows or columns first as they become more confident with recognising efficient calculations.
- They will choose jottings, informal or formal written methods depending upon which is most appropriate.
- Children should be expected to complete this for TU X TU but not for larger numbers.

<u>Adding the rows or adding the columns</u>

This should be decided by the child depending on the numbers that are produced through the calculation.

53 x 16
Estimate 50 x 20 = 1000

x	10	6
50	500	300
3	30	18

Adding the rows is the most efficient calculation:
500 + 300 = 800
30 + 18 = 48
So 800 + 48 = 848
848 is quite a distance from 1000 but rounding 53 and 16 to the nearest ten was also a significant difference. The answer is reasonable.
38 x 17

x	10	7
30	300	210
8	80	56

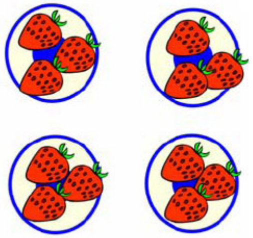
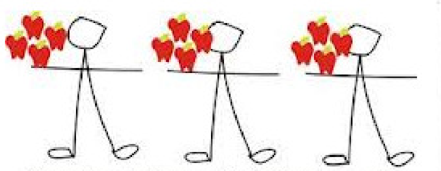
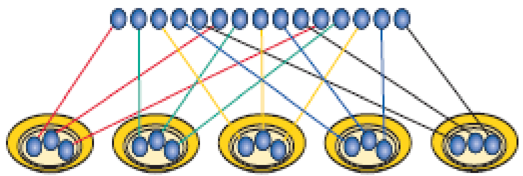
Adding the columns:
300 + 80 = 380
210 + 56 = 266
380 + 200 = 580 + 60 = 640 + 6 = 646

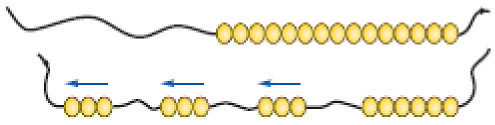
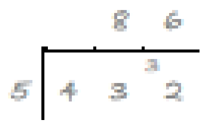
Stage 5: Expanded short multiplication (As a build up to compact to allow

Stage 5
Multiply the units first which enables them to move towards the compact method e.g.

children to see the progression from the grid method). • The first step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method above. • Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 38 × 7 is 'thirty multiplied by seven', not 'three times seven', although the relationship 3 × 7 should be stressed.	30 + 8 X 7 56 7 x 8 210 7 x 30 266	
Stage 5: Short multiplication for up to TU x 12 • The recording is reduced further, with the carried digits recorded the same as column addition, just above the top line of the answer box. • This method is appropriate for multiplying two and three digit numbers by numbers up to 12, which relies on children have recall of their times table facts up to 12.	342 × 7 becomes 3 4 2 × 7 2 3 9 4 2 1 Answer: 2394 This example shows the carried digits below the line. 124 × 12 becomes 2 4 1 2 4 × 1 2 1 4 8 8 Answer: 1 488 This example shows the carried digits at the top of the next column	

DIVISION

Stage 1: Recording and developing mental images • Children are encouraged, through practical experiences, to develop physical and mental images. • They make recordings of their work as they solve problems where they want to make equal groups of items or sharing objects out equally.	  12 apples to share with 3 friends equally	Initially recording of calculating should be done by adults to model what children have done in pictures, symbols, numbers and words. Over time there should be an expectation that children will also become involved in the recording process. Whilst cameras are an excellent way of keeping a record of what children have done, they are not a substitute for the modelling of different ways of recording calculation procedures.
Sharing and Grouping • They solve sharing problems by using a 'one for you, one for me' strategy until all of the items have been given out. • Children should find the answer by counting how many eggs **1 basket** has got. • They solve grouping problems by creating groups of the given number. • Children should find the answer by counting out the eggs and finding out how many **groups of 3** there are. • They will begin to use their own jottings to record division.	15 eggs are shared between 5 baskets. How many in each basket? First egg to the first basket, 2nd egg to the second etc  There are 15 eggs. How many baskets can we make with 3 eggs in? 	

Stage 2: Bead strings, number lines simple multiples • Using a bead string, children can represent division problems • They count on in equal steps based on adding multiples up to the number to be divided. This can be utilised for larger numbers by using number facts e.g. 4 x 5 = 20, 40 x 5 = 200. • If the problem requires 15 eggs to be **shared** between 3 baskets, the multiple of three is obtained each time all three baskets have received an egg.	15 eggs are placed in baskets, with 3 in each basket. How many baskets are needed?  Counting on a labelled and then blank number lines. 15 ÷ 3 = 5  This can also be used for numbers that require a remainder, children are encouraged to use the number line to get as close to the target number using the divisor. 44 ÷ 7 = 6 r 2 You can use the numberline to get to 42 but because you can no longer do a jump with the divisor the 2 becomes a remainder.	
Stage 3: Short division Once children have developed a sound understanding of division, using the manipulatives 'formal written methods' of short division. For calculations where numbers with up to 4 digits are divided by a single or 2 digit number, children are expected to use short division.	**Stage 3** Short division 432 ÷ 5 becomes  Answer: 86 remainder 2 With short division, children are expected to 'internalise' the working from above.	